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Unsteady Flow of Power-Law Fluids

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Introduction

SOLUTIONS for power-law fluids have been presented in Refs. 1 and 2 for both impulsively started plate and flow cases. In Ref. 1 a two-point boundary value problem was formulated and solved while in Ref. 2 a perturbation technique was used, the accuracy of which is limited to fluids which are slightly non-Newtonian. In the present Note we present a noniterative solution to the same problem for all power-law fluids formulated as an initial value rather than a boundary value problem. The formulation results in a simple expression which readily can be integrated in closed form with such forms available in integration tables.

Analysis

The differential equation pertaining to the problem has been derived before[1] and it is in the form

$$G_{\eta\eta} = -2\eta(G_\eta)^{2-N} \quad (1)$$

subject to the boundary conditions

$$G(0) = 0, \qquad G(\infty) = 1 \quad (2)$$

Let us introduce the following transformation[3] for Eqs. (1) and (2):

$$\eta = \bar{\eta}A^{\alpha_1}, \qquad G = \bar{G}A^{\alpha_2} \quad (3)$$

We obtain then the following equivalent problem:

$$\bar{G}_{\bar{\eta}\bar{\eta}} = -2\bar{\eta}(\bar{G}_{\bar{\eta}})^{2-N} \quad (4)$$

subject to the initial conditions

$$\bar{G}(0) = 0, \qquad \bar{G}_{\bar{\eta}}(0) = 1 \quad (5)$$

with

$$A = [\bar{G}(\infty)]^{-2/(N+1)}, \quad \eta = A^{(N-1)/2}\bar{\eta}, \quad \bar{G} = A^{(N+1)/2}\bar{G} \quad (6)$$

As $N = 1$ represents the Newtonian case, we analyse cases with $N \neq 1$. Integrating Eq. (4) once using the second condition in Eq. (5) yields

$$(\bar{G}_{\bar{\eta}})^{N-1} = -(N-1)\bar{\eta}^2+1$$

or

$$(\bar{G}_{\bar{\eta}}) = [1-(N-1)\bar{\eta}^2]^{1/(N-1)} \quad (7)$$

Integrating Eq. (7) using the first condition in Eq. (5) results in the following expression for $\bar{G}(\bar{\eta})$:

$$\bar{G}(\bar{\eta}) = \int_0^{\bar{\eta}} [1-(N-1)\bar{\eta}^2]^{1/(N-1)}\, d\bar{\eta} \quad (8)$$

Closed form solutions to Eq. (8) are available for a wide range of N such as $N = \frac{1}{3}; \frac{1}{2}; \frac{2}{3}; \frac{3}{4}$; 1.25; 1.5; 2; 3.

As in the work of Ref. 1 we define $\bar{\eta}_\infty$ for $N > 1$ to be the value of $\bar{\eta}$ which makes the integrand in Eq. (8) zero, i.e.,

$$1-(N-1)\bar{\eta}_\infty{}^2 = 0$$

which yields

$$\bar{\eta}_\infty = [1/(N-1)]^{1/2} \quad \text{for} \quad N > 1 \quad (9)$$

For $N < 1$, $\bar{\eta}_\infty$ is taken to be the value of $\bar{\eta}$ where $\bar{G}(\infty)$ reaches a constant asymptotic value.

Equation (8) was integrated from $\bar{\eta} = 0$ to various values of $\bar{\eta}$ including $\bar{\eta}_\infty$. The value of $\bar{G}(\infty)$ thus obtained was used to obtain the value of A and the relations between η and $\bar{\eta}$ and G and $\bar{G}$ which are given in Eq. (6). For those values of N to which closed form solutions are available, the results were obtained using such solutions. Solutions for other values of N were obtained by the numerical integration of Eq. (8).

Results and Conclusions

In Table 1 we compare the results obtained using the present method with the results presented in Refs. 1 and 2. The results compare the drag coefficient $C_f(N)$ expressed as

$$C_f(N) = [G_\eta(0)]^N/[2N(N+1)]^{N/(N+1)} \quad (10)$$

The table shows that the present method yields results with high degree of accuracy for all values of N. The results from the perturbation solution is reliable for values of N in the vicinity of 1. Furthermore, we believe that the present initial value method yields much simpler expressions in the analysis than those presented in Ref. 1 which were based upon an analysis of a boundary value problem.

Table 1 Numerical values of skin-friction coefficient $C_f(N)$

N	Present method	Ref. 1	Roy[2]	Wells[4]	Ref. 5, source of closed form solution
0.25	0.9892	0.9925	1.0099		
0.50	0.8128	0.8145	0.8219	0.816	p. 16
0.75	0.6711	0.6718	0.6727		p. 36
1.00	0.5642	0.564	0.5642	0.564	
1.25	0.4807	0.4823	0.4815		Direct analytical integration
1.50	0.4171	0.4187	0.4123		Direct analytical integration
1.75	0.3677	0.3683	0.3483		
2.0	0.3269	0.3276	0.2843		Direct analytical integration

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